Problem Set 3, Bio 4181: Due Sept. 29, 2009

1. A large ancestral population is subdivided into a large number of isolates, each of ideal size 20. An autosomal locus with two alleles, *A* and *a*, with *p* (the frequency of *A*) =0.8 in the ancestral population. Assume that genetic drift is the only evolutionary force operating.
a). What is the average allele frequency of *A* in the overall population of isolates after 5, 10, and 100 generations?
Drift has no direction, so the average allele frequency remains 0.8 at all generations. 3 pts.
b) What allele frequencies of *A* do you expect to find within the isolates at equilibrium, and what percentage of the isolates will have each specific allele frequency?
80% of the isolates will have p=1 at equilibrium, and 20% will have p=0 at equilibrium. 2 pts.
c). What is the variance in the frequency of *A* among the isolates after 10 generations of isolation?
Use equation 4.15 with p=0.8, N=20, and t=10 to get var(t=10) = 0.03579. 2 pts.
d). What is the equilibrium variance of the frequency of *A* among the isolates.
At equilibrium var = pq = (.8)(.2)= .16. 1 pts.
e). What is the average pedigree inbreeding coefficient within each isolate after 10 generations of subdivison, assuming F=0 for all individuals before subdivision.
Use equation 4.8 with p=0.8, N=20, and t=10 to get F(t=10) = 0.2237. 1 pt.

2. A large population has an allele *a* at an autosomal locus with a frequency of 0.1. The population then experiences a bottleneck and is reduced to 4 individuals.
a). What is the probability that the *a* allele will increase in frequency in the gene pool defined by these 4 individuals?
A population of 4 individuals is a sample of 8 genes, so the frequency of the *a* allele will increase above 0.1 if there are 1 or more copies of *a* in these 8 genes. It is easier to calculate the complement of this event, the probability of 0 *a* alleles. Under binomial sampling with $2N$=8 and p= 0.1 (see Statistical appendix), Prob(x=0)= $(0.9)^8$=0.4305. 2 pts.
b). The population of 4 instantly grows into an ideal population of 189 individuals and remains at that size ever after. What is the probability of ultimate fixation of the *a* allele given the initial condition that q=0.1 in the pre-bottlenecked population.
Prob. (ultimate fixation) = q =0.1 (2 points).

3. A founder population is established, and the average probability of identity by descent is calculated for each subsequent generation as follows:

Generation	**F**
1	0.005
2	0.129375
3	0.133728125
4	0.138059484

a), What are the inbreeding effective sizes in generations 1 through 4 relative to the previous generation (generation 0 is the founder population)?

Use equation 4.3 or 4.5 and solve for N to obtain:

Generation	N
1	**100**
2	**4**
3	**100**
4	**100**

2 pts for equation, and 1 pt each for each N (6 pts total)

b) What is the inbreeding effective size of each generation to the founder population?

Use equation 4.10 to obtain:

Generation	N
1	**100**
2	**7.470813064**
3	**10.70084316**
4	**13.71330443**

2 pts for equation, and 1 pt each for each N (6 pts total).

4. A set of equally sized replicate, isolated populations is established from a large population in which the frequency of the *A* allele at an autosomal locus is 0.4. The variances in allele frequency for each subsequent generation are as follows:

generation	variance
0	0
1	0.012
2	0.01314
3	0.035826
4	0.036336435
5	0.061794381

a), What are the variance effective sizes in generations 1 through 5 relative to the previous generation (generation 0 refers to the large population from which the replicates were drawn)?

From Box 4.1, you have the recursion relationship for the variance at one generation to the variance at the next. Use that to solve for the N's. (2 pts for the equation, and 1 pt for each N; 7 pts total)

generation	**N**
1	**10**
2	**100**
3	**5**
4	**200**
5	**4**

b) What is the variance effective size of each generation relative to generation 0?

Use equation 4.18 to obtain:

generation	**N**
1	**10**
2	**18.0113213**
3	**9.53061384**

4 12.43424224
5 8.650171841

2 pts for equation, and 1 pt each for each N (7 pts total).

5. A population is ideal in every way except that the size of the population fluctuates from generation to generation as follows:

generation	N
0	50
1	300
2	200
3	10
4	100
5	500

What are the approximate inbreeding and variance effective sizes of this population over the time period given above using generation 0 as the reference generation?
Use equations 4.23 and 4.25 to obtain N_{ef} = 31.14 and N_{ev} = 41.55.